The Optimal Service Levels of Final Order for Auto Service Parts

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Abstract: The importance of after-sale services is greatly emphasized as the market competition becomes fierce. Among them, the service parts not only provide product after-sale service but also play an important role in competing with competitors in the future. Especially, the supplier of service parts may no longer manufacture the parts after the certain period of time before the end of service period due to production economics. Therefore, the after-sale service providers must place a final order for the service parts to meet future demand before the final production of service parts

Hence, this study first establishes a Poisson regression model for individual service part in the end of product life cycle with declining demand. Next, a FCM method is applied to clustering service parts into different groups based on their cost and essentiality such that different service level priorities can be assigned to the groups in order to provide effective inventory management. Furthermore, the newsvendor model considering dead stock cost and lost-sale cost under the aggregate service level constraint is developed to determine the optimal order quantity for each group. Finally, we use the real data provided by a local auto company to compare our model with the current practice of this company. The results show that our model can reduce dead stock cost effectively and meets requested customer service levels for various kinds of service parts.

Keywords: Final Order; Service Level; Poisson Regression; Fuzzy C-Means; Newsvendor model

I. Introduction

With the rising of living standards and increasing customers' demanding for product satisfaction, regular logistic service from supplier to consumer or price discounts for a product can no longer improve customer satisfaction. Instead, the functionality, quality, speed to the market and after-sale service are all needed in order to compete in gaining customer's loyalty. Especially for many products which need after-sale service for longer period of time, this issue should be taken care more seriously. Take automobile industry as an example, automobile has a higher selling price, huge number of service parts, and high traffic accident rate. Thus, consumers usually take after-sale service into consideration when they shop for it. Because of the durability and high price, consumers would expect service of repairing and maintenance will cover the entire product life time. Company usually receives complaints from customers if it cannot perform the after-sale service promptly due to the unavailability of service parts. As a result, after-sale service for service parts is getting more and more important and becomes critical in today's business competition.

Due to the concern of production economics, the supplier of service parts will stop producing the service parts after a certain period of time. The service period after the end of production is called as "end-of-life service period (EOL)" [14]. Therefore, the manufacturer's final order before the end of production of service part has to be placed for EOL. But large order quantity for EOL may result in higher amount of dead stock, and small order quantity for EOL may incur customers' complaints with the result of loss sales. Also, near the end of product life time, the manufacturer may consider to maintain a minimum level of service level such that the dead stock will not become a serious problem later because the willingness of customers for repairing and maintenance service is lessen. Since the final order quantity is difficult to be remedied by other mechanisms, how to determine the optimal quantity of final order becomes a crucial issue in the practice.

In order to determine the optimal order quantity for EOL, the demand of service part with declining pattern at the end of product life cycle need to be estimated. In this study, we also adopt the remaining demand following a Poisson distribution which was adapted by most researchers to establish a more effective demand forecasting model of service parts at EOL stage. Unfortunately, thousands of service parts need to be stocked in the auto after-sale service. By determining the optimal final order size for service part individually, it will incur high administration cost. Thus, this study intends to cluster the service parts into several homogeneous groups based on their characteristics such as their essentiality and cost to simplify the decision making process and provide effective inventory management for the practice professionals. Finally, we develop a newsvendor-typed model by minimizing the total cost of dead stock and lost sale of service parts to obtain the optimal final order service level for each group. In that way, business can not only make more cost-effective final order decision but also meet customers' request up to certain service level for various types of service parts.

The rest of the paper is organized as follows. Section 2 reviews relevant literature regarding final order model for service parts. The model design and research method are presented in section 3. We verify our model with real data by performing data analysis in section 4. Finally we conclude the paper in section 5.

II. Literature Review

As mentioned above, the service period of service parts is longer than its supply period. Therefore, manufacturers must place a final order to fulfill future needs. Fortuin argued that the demand during EOL stage should decease in exponential rate when the final order issue for service parts in the end of life cycle is discussed [4]. He assumed that the demands during the EOL stage follow independent normal distributions. With the support of the ratio between average demand and standard deviation of normal distribution, he further developed the relationship among service level, length of service period, safety inventory, shortage inventory, and obsolete cost, so that the quantity of final order can be determined and the cost of shortage and out-dated stocks can be calculated. Teunter & Fortuin used actual data from Phipps Co. to classify parts into three types by their demand type to predict the optimal final order for each type [13]. Next, Teunter & Fortuin think that parts' demand is rare at EOL stage and the demand is consistent with the characteristic of a Poission distribution [14]. Therefore, they assumed demand and supply are Poission distributed and independent in each period to obtain the near-optimal solution of final order by minimizing the cost of production, holding, disposal and shortage. Hill assumed demand function as power function and derived the optimal order quantity at last period [5]. Hill et al. also assumed that the demand follows a Poisson distribution with exponential decreasing rate to obtain the optimal order quantities which minimize the cost of all the periods with dynamic programming [6]. He also proposed a newsvendor's approach to deriving the optimal solution for the final order.

Some other recent articles discussed the situation that other ways exist to continue the supply of service parts after the suppliers stop producing them. Teunter & Haneveld took the extra re-order cost after the end of production of parts into consideration, and assumed the demand as a Poisson distribution to find the optimal order level for each item [15]. Cattani & Souza further discussed the possible benefits of postponing the final order [2]. Kleber & Inderfurth considered the possibility of re-starting production associated with its cost, and obtained the optimal quantity of final order with a heuristic approach and a newsvendor model [8]. Inderfurth & Mukherjee proposed three approaches to fulfilling future demand of parts: final order, extra production or purchase, and remanufacturing and used decision tree and stochastic dynamic programming to find the best plan [7]. Van Kooten & Tan considered the situation that some parts can be repaired but others can't be recovered to be reused again. They built up a Markov model with given probability of repairing rate and time for repairing to make the final order [16].

These recent studies are different from this study in nature since this study considers multiple items and use purchasing cost and essentiality of service parts as indices which are taken into FCM algorithm and determine service level priorities for different groups. Also, this study develops a newsvendor-typed model to obtain optimal service level of final order for each group under the consideration of minimizing the costs of lost sales and dead stock.

III. Model design and research method

This section develops the final order model we propose. In the beginning, the assumptions in our model are introduced. Next, Poisson regression model and Fuzzy C-means (FCM) algorithm are proposed [1] [3]. Finally, a newsvendor-typed model is formulated to determine the optimal service level of final order for each group.

The assumptions of the models

The assumptions of our model are stated as follows.

1. We assume that the annual demand for each part follow Poisson distribution and its mean decrease in exponential rate. In addition, the demand in each period is independent. [6]
2. The demand for each part is independent.
3. Stocked parts become scraps once they exceed guaranteed service period and the salvage value of scrap parts is zero.

The Poisson regression model

First, notations are defined. Next, a Poisson regression model is developed. Finally, a statistics methods proposed by Kleinbaum et al. [9] is used to test the fitness of the Poisson regression model.

Notations for Poisson regression model

Index

i ：The index of service parts, $i = 1, 2, ..., n$

t ：The index of time (in periods), $t = 1, 2, \ldots, T, T+1, \ldots, T^e$

Input parameters

$Y_{i,t}$ ：The demand of service part *i* at period *t*

n ：The number of service parts

T ：The time that end of production of service parts

T^e ：The time that end of service period of service parts

Output parameters

$\lambda_{i,t}$ ：The mean demand of service part *i* at period *t*

a_i , b_i ：The parameter of Poisson regression model

D_i ：Demand of service part *i*

Poisson Regression model

We assume that actual demand of service part *i* at period *t* follows a Poisson distribution：

$P_r(Y_{i,t} \mid \lambda_{i,t}) = (\lambda_{i,t})^{Y_{i,t}} \exp(-\lambda_{i,t}) / Y_{i,t}!$, $Y_{i,t} = 0,1,2,.....$, $t = 1,2,.....T$ and $E(Y_{i,t}) = \lambda_{i,t} = \exp(a_i + b_i t)$

First, this study uses maximum likelihood estimation to estimate parameters a_i, b_i and applies Newton-Raphson algorithm to search the maximum likelihood estimators $\hat{a}_i, \hat{b}_i$. Next, in order to assess Poisson regression model's fit, the deviance is calculated and a chi-squared test is performed. Besides, overdispersion is needed to be tested. If the variance of sample is larger than its mean, the phenomenon is called overdispersion. This study uses heterogeneity factor (HF) to check whether overdispersion exists. The calculation is as follows: $\text{HF} = \text{Deviance}/\text{d.f.}$, where d.f. is t-2. If HF is much larger than 1, then there exists the risk of overdispersion. According to Kleinbaum et al. [9], HF between 1.5 to 2 is acceptable.

Demand distribution of each part for remaining service periods

According to Poisson regression model, we can calculate average demand of parts for remaining service periods. This model presumes that the demand of each part in each period follows Poisson distribution and is independent. Consequently, the mean, variance, and the distribution of part's demand for remaining service periods can be derived as follows:

$$E(D_i) = \sum_{t=T+1}^{T^e} \lambda_i(t) = \sum_{t=T+1}^{T^e} \exp(a_i + b_i t) = \lambda_i \tag{1}$$

$$Var(D_i) = E(D_i) = \lambda_i \tag{2}$$

$$D_i \sim \text{Poisson}\left(\sum_{t=T+1}^{T^e} \lambda_i(t) = \sum_{t=T+1}^{T^e} \exp(a_i + b_i t) = \lambda_i\right) \tag{3}$$

Clustering model under service differentiation

This section illustrates how to cluster a large number of service parts into several groups such that service level priorities can be determined later based on the importance of these groups in order to provide efficient inventory management. The details of FCM clustering process are as follows.

Step 1: Choosing clustering index

There are two main factors for customers to decide whether to replace parts or not when the product is in the declining stage of life cycle. The first factor is essentiality of the parts. The second factor is cost of the parts.

We presume that essentiality is the average demand of single part for remaining service periods divided by the demand of all parts for remaining service periods. This definition can represent the degree of willingness of customers to replace the service part which also indicates the essentiality of the part indirectly. The calculation of essentiality as equation (4)

$$\text{Essentiality} = \sum_{t=T+1}^{T^e} \lambda_i(t) \Big/ \sum_{i=1}^{n} \sum_{t=T+1}^{T^e} \lambda_i(t) = \lambda_i \Big/ \sum_{i=1}^{n} \lambda_i \tag{4}$$

The reason of service level differentiation by grouping is to simplify the decision making process for thousands of parts and obtain reasonably good results. Hence, we cluster parts into homogeneous groups based on their essentiality and cost. Figure 1 indicates the basic idea of clustering diagram.

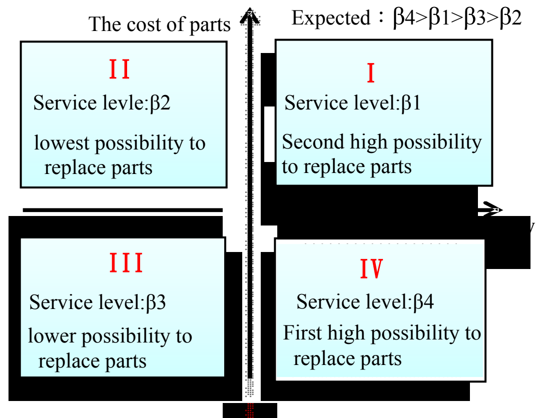

Figure 1 [redacted]ering diagram

Step 2: Normalizing data

Before the data are clustered, the data set needs to be screened to avoid the impact of measure unit. The equation 5 is used to get the normalized value for each service part [17].

$$\text{Normalize value} = \frac{(0.9-0.1)(\text{Original value} - \text{Min})}{\text{Max-Min}} \tag{5}$$

Before the clustering process is further explored, some notations used in this process will be introduced.

i：The index of group，$i = 1,2,...,c$

j：The index of service parts，$j = 1,2,...,m$

E_j ：The essentiality of service part j

P_j ：The cost of service part j

E_j^w ：Normalized value of the E_j

P_j^w ：Normalized value of the P_j

$x_j = [E_j, P_j]$ ：Vector of service part j

$W_j = [E_j^w, P_j^w]$ ：Normalized vector of x_j

l：The power of membership function

c：Pre-determined number of group($2 \le c \le N$)

k：The maximum allowable convergence runs。

ε ：Convergence criterion variable

$U_{c\times m}$ ：Membership function

u_{ij}： The element of $U_{c\times m}$, $i = 1,2,...,c$, $j = 1,2,...,m$

c_i ：The centroid of group I, $i = 1,2,...,c$

Step 3: Building up membership function $U^{k=0}$

The possibility of data points belongs to which group is determined by membership function $U_{c\times m}$. We can randomly generalize a matrix $U_{c\times m}$.

$$U_{c\times m} = [u_{ij}],\ i = 1,2,\ldots,c,\ j = 1,2,\ldots,m \tag{6}$$

Data point x_j belongs to the group which has the highest membership corresponding value.

Step 4: Finding the centroid of each group C^k

After the membership function determined, equation 7 is used to find the centroid of each group $C^k = \{c_i \mid i = 1, 2, \ldots, c\}$.

$$c_i = \frac{\sum_{j=1}^{m} u_{ij}{}^{l} x_j}{\sum_{j=1}^{m} u_{ij}{}^{l}} \tag{7}$$

Step 5: Calculating the objective function

After determining the centroid of each group, we can use equation 8 to calculate objective function. The smaller the value of the objective function, the better the result of clustering.

$$J(U, c_1, c_2, \ldots, c_c) = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c}\sum_{j=1}^{m} u_{ij}^{l} \left\| x_j - c_i \right\|^2 \tag{8}$$

$$\| x_j - c_i \|^2 = (x_j - c_i)^T (x_j - c_i),\ \forall j, i$$

Step 6: Finding a new membership function U^{k+1}

By substituting $C^k = \{c_i \mid i = 1, 2, \ldots, c\}$ into equation 9, we are able to find a new membership function.

$$u_{ij} = \frac{1}{\sum_{r=1}^{c} \left(\frac{\| x_j - c_i \|}{\| x_j - c_r \|} \right)^{\frac{2}{l-1}}} \tag{9}$$

Step 7: Checking the convergence conditions

The convergence criterion is shown as equation 10. If $\|J^{k+1} - J^k\|$ is larger than ε and the total runs are less than *k*, go back to step 3. On the contrary, if $\|J^{k+1} - J^k\|$ is less than ε or the total runs exceed *k*, we stop the FCM clustering process.

$$\|J^{k+1} - J^k\| < \varepsilon \tag{10}$$

Step 8: Determining the optimal number of groups

This study applies the XB Index [18] to determining the optimal group number. The calculation of XB index is shown as equation 11.

$$I_{XB}(U, C; X) = \frac{\sum_{i=1}^{c}\sum_{j}^{m} u_{ij}{}^{2} \| x_j - c_i \|^2}{m * (\min_{i \neq r} \| c_i - c_r \|^2)} \tag{11}$$

The smaller the XB index is, the better the cluster result is. To make the optimal number of groups become more robust, this study repeats the loops in 30 times for different *c (2~N)*. Every time we can obtain the minimum value of XB index and its corresponding optimal number of groups which is the suggesting input of group number for next loop. Finally, we assign the original data points to the different groups according to their membership function.

The final order model

We use centroid demand in each group to represent the demand of all parts in the same group in order to simplify the number of data points required by the input of newsvendor model. After the optimal final order quantity and service level for each group are obtained, the orders of the parts in the same group can be placed based on these optimal values. This reduces the frequency of order and its complexity. Besides, manager can enforce the service level priorities based on company's strategy or add a minimum total service level needed to satisfy customer request in our final order model. The details of model formulation are as follows:

Notations for newsvendor model

Index

i : The index of group, $i = 1, 2, ..., c$

j : The index of service part j, $j = 1, 2, ..., m$

t : The time index (in periods), $t = 1, 2, \ldots, T, T+1, \ldots, T^e$

Input parameters

c : The optimal number of groups, $c \in N$

λ_i : The average centroid demand of group *i* for remaining service periods

T : The time that end of production

T^e : The time that end of the service period

Co_i : Dead stock cost of group *i*

Cu_i : lost-sale cost of group *i*

D_i : Demand of group *i* from period $T+1$ to period T^e

Decision variable

S_i : Final order quantity of group *i*

Objective function

TC : Total cost, including the dead stock cost and lost-sale cost

Demand distribution of each group for remaining service periods

As mentioned above, we get centroid coordinates for each group c_i, $c_i = [E_i^w, P_i^w]$ after clustering a larger number of parts into *i* groups by FCM method. In addition, the formula of essentiality is shown as equation 12.

$$E_i = \lambda_i / \sum_{j=1}^{m}\sum_{t=T}^{T^e} \lambda_j(t) \tag{12}$$

Therefore, we can obtain average centroid demand λ_i by transforming E_i^w back to E_i (the value before performing the normalization) and then replacing E_i in equation 12 to calculate the average centroid demand. As a result, the demand distribution of group *i* for remaining service periods D_i follows a Poisson distribution. $D_i \sim Poisson(\lambda_i)$.

Cost of lot-sale and dead stock

Sales will be lost when the inventory stocks out. Thus, cost of lost-sale can be calculated as subtracting the initial purchasing cost from the sale price. Since holding cost and salvage value are assumed to be negligible in our model, stocking parts become scraps once the service periods have ended. In other words, cost of dead stock is the purchasing cost of a service part. Finally, the average purchasing cost of group *i* which represents the dead stock cost of group *i* (*Co*) can be obtained by reversing the normalization process of c_i, and the average lost-sale cost of group *i* which represents by the lost-sale cost of group i (Cu) can also be obtained by subtracting the average purchasing cost of group i from the average selling price of group i.

Optimal order quantity and service level for each group

Once the costs of lost-sale and dead stock are figured out, the newsvendor-typed model is ready to be applied to our study. Notice that fixed ordering cost and holding cost are not included in our model. Also, no backorder or rush ordering or remanufacturing cost are not considered in our scenario. Thus, the total cost function is developed as follows:

$$TC = \sum_{i=1}^{c}(Co_i \sum_{d_i=0}^{S_i} \frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}(S_i - d_i) + Cu_i \sum_{d_i=S_i}^{\infty} \frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}(d_i - S_i)) \tag{13}$$

Since the total cost function can be separated into disjoint cost function of different groups, the optimal quantity of final order for each group can be derived based on marginal analysis.

$$F(S_i^*) = \frac{Cu_i}{Cu_i + Co_i} \tag{14}$$

$$S_i^* = F^{-1}\left(\frac{Cu_i}{Cu_i + Co_i}\right), \ \frac{Cu_i}{Cu_i + Co_i} \text{ is called critical ratio} \tag{15}$$

However, a Poisson distribution is a discrete distribution and the optimal order quantity S_i^* should be a positive integer not a continuous real number. As a result, we can find the optimal order quantity S_i^{**} by which a smallest integer value that has a Poisson cumulative function value $F\left(S_i^{**}\right)$ exceeds critical ratio. By substituting this optimal order quantity back to the total cost function, we can obtain the minimum total cost.

As for service level, this study chooses β-service-level to examine the service level [11] [12]. It is defined as follows:

$$\beta = 1 - [E(\text{Backorders per period})/E(\text{Period demand})] \tag{16}$$

According to the formula of β-service-level, we can obtain the optimal service level for each group β_i^* and the optimal aggregate service level β^* .The formulas are shown in equation 17 and 18

$$\beta_i^* = 1 - \frac{1}{\lambda_i}\sum_{d_i=S_i}^{\infty}\frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}(d_i - S_i^*) \tag{17}$$

$$\beta^* = \sum_{i=1}^{c}\frac{\lambda_i}{\lambda}[1 - \frac{1}{\lambda_i}\sum_{d_i=S_i}^{\infty}\frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}(d_i - S_i^*)] \ ; \ \lambda = \sum_{i=1}^{c}\lambda_i \tag{18}$$

The case with the constraints of aggregate service level or prioritized service level

If the manufacturer finds that the aggregate service level deriving from our optimal solution do not meet the requirement or the priority for different groups in terms of service level obtaining from our optimal solution is different from managers' expectation which demonstrates in section 3.3, we can add either constraints (19) or (20) to our newsvendor model.

$$\beta^* = \sum_{i=1}^{c}\frac{\lambda_i}{\lambda}[1 - \frac{1}{\lambda_i}\sum_{d_i=S_i}^{\infty}\frac{e^{-\lambda_i}\lambda_i^{d_i}}{d_i!}(d_i - S_i^*)] \geq \text{Planning service level} \tag{19}$$

$$\beta_i > \beta_j \ , \ i \neq j \tag{20}$$

Constraint 19 represents the aggregate service level must exceed the planned service level. Constraint 20 represents the service level of group i must larger than the service level of group j if priority of group i is higher than group j.

IV. Data analysis and discussion

In this section, we verify our model with real data provided by a local auto company by performing data analysis. In order to do that, we first introduce the current practice of the auto company. Next, we build up parts demand forecasting model by using a Poisson regression model. Furthermore, a FCM method is applied to cluster service parts into different groups and the optimal final order quantity for each group is calculated by using a newsvendor-type approach. Finally, our results will be compared with current practice of the auto company

The current practice of the auto company

Currently, the local Auto firm has a logistics center to take the responsibility for after-sale service parts in Taiwan. The logistics center has two different sources of supply for service parts: one is from domestic suppliers; the other is from overseas suppliers. They have different supply policies. For the domestic suppliers, the supplier would stop producing the parts, and ask for the auto company to place its final order if the car has been stopped manufacturing for 10 years and the average annual demand is less than 40 units in the past 3 years, or no demand in the past 3 years. At that moment, the auto company has to decide the final order quantities to satisfy future demands. If the order quantities were insufficient, the shortage occurs. On the other hand, the dead stock may exist if the parts can't be sold out after all. For the overseas suppliers, that is not the case in general, and parts in shortage can be transported by air with rush orders. Only when the global annual demand is less than 40 units, the overseas suppliers will ask for last buy.

Once the parts demand meets the above specifications; the domestic suppliers would stop producing the parts. The Auto firm will place its final order according to (21):

$$\text{Final order quantities} = (1/2) \times \text{the average year demand in past three years} \times \text{remaining service periods} \tag{21}$$

Predicting the demand of service parts with a Poisson regression model

There are 58 major service parts with nearly zero demand in this auto company during the periods from 2005 to 2008. The parts can be divided into three different patterns based on their annual demand data. Type-1 parts have the property that the demand continues declining in each year after car stops producing (immediate decline pattern). Type-2 parts show that annual demand increases in the beginning and then decreases afterward when car stops producing (delay decline pattern). Type-3 parts have low demand and little fluctuation in each year after car stops producing (slow moving pattern).

Demand patterns of Type-1 and Type-3 parts can be directly fitted by using Poisson regression model. $\lambda_i(t) = \exp(a_i + b_i t)$ $b_i < 0$, $t = 1,2,....,T$, i=1,2,…58 Nevertheless, the demand pattern of type-2 parts which increases in the beginning and then decreases afterward may result in positive parameter ($b > 0$) if the Poisson regression model is used. Therefore, this study use the method suggested by Moore [10] to build up a Poisson regression model without violating the assumption ($b < 0$).

The results show that most of the heterogeneity factors (HF) of the service parts are around 1.5, and only 4 service parts are larger than 3. Therefore, the most of data do not appear to have serious overdispersion problem. In addition, from the p-value of the test for model fit and parameter *b*, we can not reject null hypothesis H_0 : the data fit a Poisson regression model at the 5% significance level.

Clustering service parts

We choose the power of membership function l = 2, the maximum acceptable number of group N is 10 ($2 \le c \le N$), the maximum number to convergence k is 300 and ε is 0.00001.

According to the aforementioned settings and repeat 30 times for loop operations, different group number *c* and its corresponding XB index is shown in table 1. The results show that the minimum value of XB index is at the group number 3. As a result, this study chooses *c*=3 as the optimal group number.

Table 1 Different group and its corresponding XB Index

c	2	3	4	5	6
XB-Index	0.1698	**0.0542**	0.1485	0.4497	0.2814
c	7	8	9	10	
XB-Index	0.2635	0.6473	0.2641	0.4845	

Since the optimal group number is 3, the result of clustering can be shown in Figure 2. It can be observed that group one (□) has the higher value of essentiality, and group two (○) and group three (*) have relative lower value of essentiality. In addition, the average cost of group two is lower than that of group three. At the time when suppliers stop producing service parts, auto should be old enough. Hence, whether the customer will replace the parts depends on the essentiality of parts. If the essentiality of parts is relatively low, the customers will not replace the parts. Furthermore, customers will choose cheaper parts to replace under the same level of essentiality. As a consequence, we suggest that the service level of group one should be the highest. On the contrast, the service level of group three is the lowest.

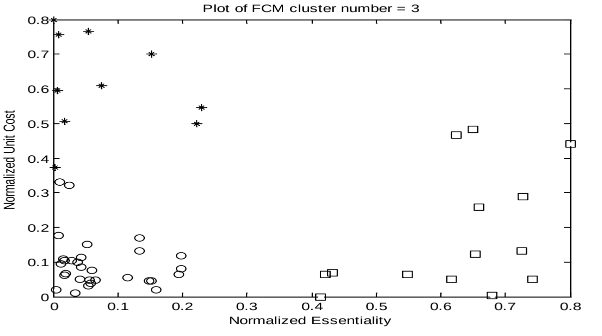

Figure 2 Results of cluster diagram

Determining the optimal quantity of final order for each group

After three groups are formed, the average centroid demand, average sale price and centroid purchasing cost of parts can be calculated. The computational results are shown in Table 2. Next, the optimal order quantities of these three groups can be determined by our newsvendor model. It turns out the optimal order quantity for the service parts of these 3 groups are 45, 6, and 6, respectively. Finally, we derive the optimal service level β_i^* for each group. They are $\beta_1^* = 0.9438\%$, $\beta_2^* = 0.8825\%$, and $\beta_3^* = 0.8607\%$ respectively and the aggregate expected service level β^* is 0.9295.

From the results, the service level of group 1 is the highest, the service of group 2 is the second highest, and the service level of group 3 is the lowest. We find the preference of service levels for 3 groups is the same as our expectation; therefore, we do not need to readjust the service level for groups. As a result, the auto company can attain the minimum cost under the condition that the expected service level in each cluster is 94%, 88%, and 86% and the expected aggregate service level is about 93%.

The comparison between current practice and our approach

The comparison results of both approaches are shown in Table 3. The current practice tends to create large final order which results in higher cost of dead stock and it indicates the current practice fails to catch the pattern of demand and ignores the impact of overstocking cost. The results also show that our approach still can keep the actual overall service level close to 88% service level foe all service parts. Compared to current practice, although our approach results in little higher shortage, the inventory and total cost can maintain at a lower level. Therefore, we can reduce the inventory level and total cost significantly in our approach is still lower than that in current practice.

Table 2 Centroid demand, sale price and centroid cost of groups

Group	The number of parts in a group	Average centroid demand(λ_i)	Average sale price	Centroid purchasing cost of groups	Shortage cost of groups
1(□)	14	44.6884	517.3682	230.8132	287.8
2(○)	30	5.3246	348.4246	121.2443	227.2
3(*)	10	5.6684	2148.2847	844.5847	1303.7

Table 3 The comparisons between current practices with our approach

	Inventory	Shortage	Dead stock cost	Lost-sale cost	Total cost	Actual overall service level
Current practice	260	82	80059	25129	105188	89.4%
Our approach	188	96	52712	36369	89081	87.7%

V. Conclusions

In conclusion, this research focus on the service parts at EOL stage to discuss the final order problem. We establish a Poisson regression model to predict the demand of parts. In addition, a FCM method is applied to cluster parts and we determine service level priority to provide effective inventory management. Next, we combine the results of demand predicting and clustering results with newsvendor model to obtain the optimal final order quantity and service level for each group. The orders of the parts belong to the same group will be placed based on the optimal order quantity for this group in order to reduce the complexity of order.

Finally, according to the results, we conclude that: (1) our approach considers the declining pattern of demand in predicting future demand, which prevents to result in a large order size and also increases the dead stock cost; (2) our approach provides a way to balance between total cost and customer satisfaction by clustering service parts and determining service level priority; (3) the approach we propose can determine the final order quantity which efficiently improves the current situation of high dead-stock cost resulting from decreasing pattern of demand in the end of life cycle.

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